9/5 Recitation

In this recitation, you will need the commands found in the **Julia4Systems.ipynb** notebook.

1. Use Gaussian elimination to put each system in upper triangular form and determine how many solutions each systems has. If a system has a unique solution, use backward substitution to find the solution; verify your calculation with Julia (beware of rounding errors).

a) $$\begin{aligned} x + 2y - 2z &= -15 \\ 2x + y - 5z &= -21 \\ x - 4y + z &= 18 \end{aligned}$$

b) $$\begin{aligned} x + 2y - 2z &= -15 \\ 2x + y - 5z &= -21 \\ -x - 2y + 2z &= 14 \end{aligned}$$

c) $$\begin{aligned} x + 2y - 2z &= -15 \\ 2x + y - 5z &= -21 \\ -x - 2y + 2z &= 15 \end{aligned}$$

2. For what values of k does the following matrix have three pivots?

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ 2 & 2 & 2k \end{bmatrix}$$

3. Which of the following matrices are in RREF? For the matrices not in RREF, describe the row operations needed to put them in RREF.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

4. **Looking Ahead:**
Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$. Solve the matrix equations $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $A\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, where $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$. Let $\mathbf{a}$ be the solution to the first equation and let $\mathbf{b}$ be the solution to the second equation. Let $B = \begin{bmatrix} \mathbf{a} & \mathbf{b} \end{bmatrix}$ be the 2×2 matrix whose columns are the solutions to the above matrix equations. Show that $AB = I$ and $BA = I$.[1]

5. **Just For Fun**: Geometry of Higher Dimensions. An equation in two variables can be represented by a line in $\mathbb{R}^2$; two such lines can intersect in a point. An equation in three variables can be represented by a plane in $\mathbb{R}^3$; two such planes can intersect in a line. How can one represent an equation in four variables? What does the intersection of two such objects look like?

[1] A and B are called *inverses* of each other. We will discuss inverses tomorrow.